

Decoherence of Highly Mixed Macroscopic Quantum Superpositions

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It is known that a macroscopic quantum superposition (MQS), when it is exposed to environment, decoheres at a rate scaling with the separation of its component states in phase space. This is more or less consistent with the well known proposition that a more macroscopic quantum state is reduced more quickly to a classical state in general. Effects of initial mixedness, however, on the subsequent decoherence of MQSs have been less known. In this paper, we study the evolution of a highly mixed MQS interacting with an environment, and compare it with that of a pure MQS having the same size of the central distance between its component states. Although the decoherence develops more rapidly for the mixed MQS in short times, its rate can be significantly suppressed after a certain time and becomes smaller than the decoherence rate of its corresponding pure MQS. In an optics experiment to generate a MQS, our result has a practical implication that nonclassicality of a MQS can be still observable in moderate times even though a large amount of noise is added to the initial state.

I. INTRODUCTION

The behaviors of microscopic quantum systems are radically different from our everyday experience in the macroscopic world. The superposition principle of quantum mechanics plays the crucial role for counter-intuitive behaviors of quantum objects. Since Shrodinger's famous illustration of a macroscopic object in a quantum superposition [1], there have been great interests in manipulating and observing macroscopic quantum superpositions (MQSs) [2-6]. However, decoherence is known as the main obstacle to this attempt [7]. It is well known that a MQS loses its quantum properties, through its interaction with environment, much faster than a microscopic one [7,8]. In this context, decoherence is often used to explain how the classical world appears from the microscopic entities which individually obey quantum mechanical principles [7].

A superposition of two coherent states that are distinctly separated in phase space is a well known example of a MQS [9,10]. This pure MQS shows nonclassical properties such as negativity of the Wigner function and interference fringes in phase space [10]. Remarkably, it has been proved that separation between the component states of a MQS is a crucial factor determining its decoherence rate [11-13]. The more separated in the phase space the component coherent states are, the faster the quantum features disappear. A similar trend is also confirmed in the decoherence of Einstein-Podolsky-Rosen (EPR) correlations [14]. It is more or less consistent with the well known observation that more macroscopic quantum superpositions decohere faster in general [7]. In practical situations, the quantum system is not always in a pure state initially, but can be in a certain mixed state by various experimental imperfections. However, effects of initial mixedness on the subsequent deco-

herence of MQSs have not been studied in detail even though truly macroscopic physical systems are typically in mixed states.

Recently, Jeong and Ralph showed that a *highly mixed* MQS can also exhibit strong quantum properties even as its entropy (i.e. mixedness) becomes extremely large [15]. One such example is a superposition of thermal states separated in the phase space [15]. This result raises another related, interesting, questions concerning decoherence of MQSs: Which state is more robust against decoherence between a highly mixed MQS and a pure MQS when their component states are equally separated in the phase space? Also, which state is more robust against decoherence when they are equally macroscopic, i.e., the sizes of the physical systems are equal? More generally, how could mixedness affect the evolution of decoherence in MQSs?

In this paper, we show that nonclassicality of a highly mixed MQS can disappear more slowly after a certain time than that of a pure MQS when the centers of their component states are equally separated. This implies that initial mixedness may not be very detrimental to the observation of nonclassical properties for a MQS in moderate times. For example, in an optics experiment to generate a MQS, nonclassicality of a MQS can be still observable even though the Gaussian noise is inevitably added to the initial state. We also make a similar comparison between a pure MQS and a mixed MQS that are "equally macroscopic" in the sense that their average photon numbers are the same. In this case, nonclassicality of a mixed MQS disappears conspicuously slower than that of a pure MQS. These two observations are explained by the fact that the initial mixed MQS can be decomposed into a sum of pure MQS's, some of which are more robust against decoherence than the single pure MQS. Our results provide a practical basis for observing fragile macroscopic quantum phenomena, and imply that

the total energy quanta may not be a good indicator of decoherence rate beyond short-time regime.

II. PURE AND MIXED MACROSCOPIC QUANTUM SUPERPOSITIONS

We first introduce the pure MQS [9,10]. A coherent state, $|\alpha\rangle$, when its amplitude α is large, is known as most classical among all pure states [16]. A superposition of two coherent states (SCS),

$$|\Psi_\alpha\rangle = \mathcal{N}_\alpha (|\alpha\rangle - |-\alpha\rangle), \quad (1)$$

where $\mathcal{N}_\alpha^{-1} = \sqrt{2 - 2e^{-2|\alpha|^2}}$, is considered a MQS for $|\alpha| \gg 1$ [17]. Very recently, such SCSs were experimentally generated in free-traveling fields [18]. We stress that the SCS in Eq. (1) may be considered a MQS only if the amplitude α is sufficiently large. In this regime, the average photon number of the state is very large and the two coherent states, $|\alpha\rangle$ and $|-\alpha\rangle$, are macroscopically distinguished. We shall suppose this condition throughout the paper.

Now, let us consider a different type of MQS with initial mixedness as follows. A coherent state of amplitude α , when exposed to a Gaussian noise with variance V , is represented by

$$\rho^{th}(V, \alpha) = \int d^2\beta P_\beta^{th}(V, \alpha) |\beta\rangle\langle\beta| \quad (2)$$

where

$$P_\beta^{th}(V, \alpha) = \frac{2}{\pi(V-1)} \exp\left[-\frac{2|\beta-\alpha|^2}{V-1}\right]. \quad (3)$$

A mixed MQS with sufficiently large α can be represented as

$$\rho = \mathcal{N} \left(\rho^{th}(V, \alpha) + \rho^{th}(V, -\alpha) - \sigma(V, \alpha) \right), \quad (4)$$

where $\sigma(V, \alpha) = \int d^2\beta P_\beta^{th}(V, \alpha) |\beta\rangle\langle-\beta| + H.C.$ and $\mathcal{N} = (2 - 2 \exp[-2\alpha^2/V]/V)^{-1}$. When $V = 1$, the state ρ becomes a pure MQS in Eq. (1). It was shown that this mixed MQS can be generated when a displaced Gaussian state in Eq. (2) is used as the input state, instead of a pure coherent state, to the SCS generation process [15]. The mixed state (4) shows strong nonclassical properties regardless of the values of α or V [15]. Note that quantum behaviors of the MQS are due to the coherence term $\sigma(V, \alpha)$. If $\sigma(V, \alpha)$ were zero, the mixed state (4) would become a mere classical mixture of the two local Gaussian states without any quantum properties.

Mixedness of a state ρ can be quantified by its linear entropy $S(\rho) = 1 - \text{Tr}(\rho^2)$. The degree of mixedness of the state in Eq. (4) is given by

$$S(\rho) = 1 - 4(\mathcal{N})^2 \left(\frac{1 + \exp[-\frac{\alpha^2}{V}]}{V} - \frac{4 \exp[-\frac{4\alpha^2 V}{1+V^2}]}{1+V^2} \right). \quad (5)$$

We shall say that state ρ is *highly mixed* when $S(\rho) > 0.99$.

The Wigner function of a quantum state is a quantum mechanical analogy of the probability distribution in phase space [19,20], and it can take negative values unlike classical probability distributions. This negativity is regarded as a clear signature of nonclassicality of a physical system [21]. In order to experimentally observe negative values of the Wigner function, which we shall often call simply “negativity” in this paper, the size of negativity must be large enough. This is particularly true in experiments with limited detection efficiency. For example, homodyne detection in quantum optics can be used to reconstruct the Wigner function. However, the efficiency of homodyne detection cannot be perfect in real experiments and small negative values of Wigner functions are hard to directly observe.

III. DECOHERENCE

It was pointed out that the decoherence rate of pure MQS scales with the distance between the component states [11,12]. A similar trend is also confirmed in the decoherence of quantum nonlocality for EPR states in the thermal environment [14]. In Ref.[14], Jeong *et al.* showed that the more strongly the initial field is squeezed (i.e. closer to the ideal EPR state), the more rapidly the maximum nonlocality decreases. Note that in the limit of the ideal EPR state, the average photon number at each mode approaches infinity. In particular, the authors of Ref.[14] explained the scaling of vanishing nonlocality with the degree of squeezing as follows. An EPR state can be understood as a multi-mode superposition of two-mode coherent states. As the degree of squeezing becomes larger, the superposition between component coherent states extends further so that the average separation between the component states becomes larger. This causes the quantum coherence, more precisely, quantum nonlocality, to be destroyed more rapidly.

In this paper, we compare the decoherence of the mixed MQS (actual state) in Eq. (4) with that of the pure MQS (target state) in Eq. (1) for the same values of α , namely, under the condition that the central distances between their component states are equal. In addition, we also look into the evolutions of the two MQSs, one pure and the other mixed, at the same levels of “macroscopicity”. For this purpose, the average photon number of MQS is adopted as the size of macroscopicity. The average photon number \bar{n} of the mixed MQS (4) is

$$\bar{n} = \text{Tr}[\hat{a}^\dagger \hat{a} \rho] = \frac{\mathcal{N} \exp[\frac{2\alpha^2}{V-1}] \{V(V-1)Q_{(5)} + 2\alpha^2 Q_{(7)}\}}{V^3(V-1)} \quad (6)$$

where \hat{a}^\dagger (\hat{a}) is the creation (annihilation) operator and $Q_{(n)} = \exp[-\frac{2\alpha^2}{V-1}] V^{(n/2)} (V-1) \sqrt{V+V^{-1}-2}$. Note that the average photon number of the pure MQS is $\bar{n} = \alpha^2 \coth[\alpha^2]$.

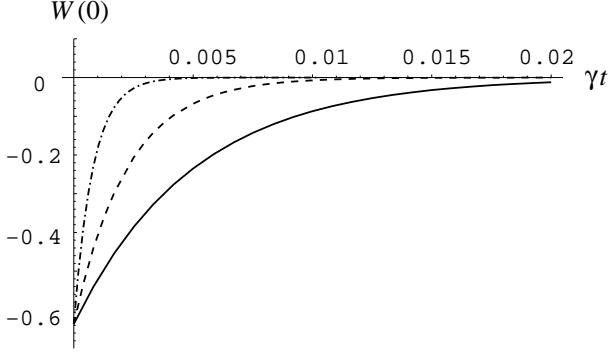


FIG. 1: The minimum negativity of the Wigner functions, $W(0)$, of pure MQSs for $\alpha = 20$ (solid curve), $\alpha = 30$ (dashed curve) and $\alpha = 50$ (dot-dashed curve) against the scaled time γt . The minimum negativity of a MQS approaches zero more rapidly when α is larger.

The decoherence of the MQSs can be studied by solving the master equation [22]

$$\frac{\partial \rho}{\partial t} = \hat{J}\rho + \hat{L}\rho; \quad \hat{J}\rho = \gamma a\rho a^\dagger, \quad \hat{L}\rho = -\frac{\gamma}{2}(a^\dagger a\rho + \rho a^\dagger a) \quad (7)$$

where γ is the energy decay rate and t is time. In the above equation, we assume the optical-frequency regime in which the average photon number in the reservoir is negligible even at room temperature. The well known solution of the master equation for a coherent-state dyadic $|\alpha\rangle\langle\beta|$ can be described as

$$|\alpha\rangle\langle\beta| \rightarrow e^{-(1-\kappa)\{(|\alpha|^2+|\beta|^2)/2-\alpha\beta^*\}}|\sqrt{\kappa}\alpha\rangle\langle\sqrt{\kappa}\beta|, \quad (8)$$

where $\kappa \equiv e^{-\gamma t}$. Using this solution, the superposition of thermal states after time t is given by

$$\rho(t) = \mathcal{N}(t) \left(\rho^{th}(V', \alpha') + \rho^{th}(V', -\alpha') - \sigma^C(V', \alpha') \right), \quad (9)$$

where

$$\sigma^C(V', \alpha') = \int d^2\beta P^{th}(V', \alpha') e^{-2(1-\kappa)|\beta|^2} |\beta\rangle\langle-\beta| + \text{H.C.}, \quad (10)$$

$\alpha' = \sqrt{\kappa}\alpha$, $V' = \kappa(V-1) + 1$, and

$$\mathcal{N}(t) = 2 - \frac{8 \exp[-\frac{2\alpha^2(3\kappa+1)}{3\kappa(V-1)+(V+3)}]}{3\kappa(V-1)+(V+3)}. \quad (11)$$

The Wigner function $W(\eta)$ of a density operator ρ can be obtained as

$$W(\eta) = \frac{1}{\pi^2} \int d^2\xi e^{\eta\xi^* - \eta^*\xi} \chi(\xi) \quad (12)$$

where $\chi(\xi)$ is the Weyl characteristic function $\chi(\xi) = \text{Tr}[D(\xi)\rho]$ for the density operator ρ with $D(\xi) = \exp[\xi\hat{a}^\dagger - \xi^*\hat{a}]$. We have found that the Wigner function of the MQS in Eq. (4) shows the minimum negativity at

the origin of the phase space, $\eta = 0$, at all times of its evolution. Therefore, to characterize the nonclassicality of the MQS, we focus on the value of the Wigner function $W(\eta = 0)$, which is essentially the photon-number parity of the state [23]. It is given by

$$W(0) = \frac{4 \left[\frac{e^{-\frac{2\alpha^2\kappa}{A}}}{A} - \frac{4e^{-\frac{2\alpha^2(1-\kappa)}{B}}}{B} \right]}{\pi \left[2 - \frac{8e^{-\frac{2\alpha^2(1+3\kappa)}{C}}}{C} \right]} \quad (13)$$

where $A = \kappa(V-1) + 1$, $B = -\kappa(V-1) + (V+3)$, $C = 3\kappa(V-1) + (3+V)$. When $\gamma t = 0$, the minimum negative values of the Wigner functions of the pure and the mixed MQSs are $-2/\pi$ (≈ -0.64) regardless of α and V .

We plot the minimum negativity of the Wigner function, $W(0)$, of pure MQSs as a function of time in Fig. 1, where evolutions of pure MQSs for $\alpha = 20$, $\alpha = 30$ and $\alpha = 50$ have been compared. As was pointed out in Ref. [11-13], it is obvious that the negativity approaches zero faster with the larger amplitude α , i.e., when the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are more separated in the phase space.

We now compare the negativities of pure MQSs and of mixed MQSs under the condition that their component states are equally separated. In Fig. 2(a), the pure MQS of $\alpha = 30$ are compared with a mixed MQS of $V = 1000$ and $\alpha = 30$. Note that the mixed MQS is highly mixed as $S(\rho) \approx 0.999$. As shown in the figure, the negativity of the mixed MQS reduces more rapidly at the initial stage and thus negativity of the pure state is deeper for $t < 0.0025$. However, Fig. 2(a) shows that the negativity of the mixed MQS remains deeper for $t > 0.0025$ and reduces slower than that of the pure MQS. In Fig. 2(b), where a similar trend is observed, the pure MQS of $\alpha = 100$ is compared with the mixed MQS of $V = 10^4$ and $\alpha = 100$, of which degree of mixedness is $S(\rho) \approx 0.9999$.

We also make comparison of decoherence between pure and mixed MQSs that are equally macroscopic in view of the average photon number. In these cases, the component states of the mixed MQSs should be less separated than those of the pure MQSs in the phase space. In Fig. 3(a), a pure MQS with $\alpha = 30$ and a highly-mixed MQS with $V = 10^3$ and $\alpha = 20$ are compared, where the average photon number is $\bar{n} \approx 900$ for both states. Here, the degree of the mixed MQS is $S(\rho) \approx 0.999$. Figure 3(a) clearly shows that the mixed MQS decoheres more slowly. In Fig. 3(b), a pure MQS with $\alpha = 100$ and a highly-mixed MQS ($S(\rho) \approx 0.9999$) with $V = 1.5 \times 10^4$ and $\alpha = 50$ are compared, where the average photon number is all $\bar{n} \approx 1.0 \times 10^4$. The same trend is also manifest in Fig. 3(b).

At first sight, the above results might look somewhat counter-intuitive, as one usually expects that the mixedness plays a negative role as degrading quantum effects. These findings can be accounted for by carefully rear-

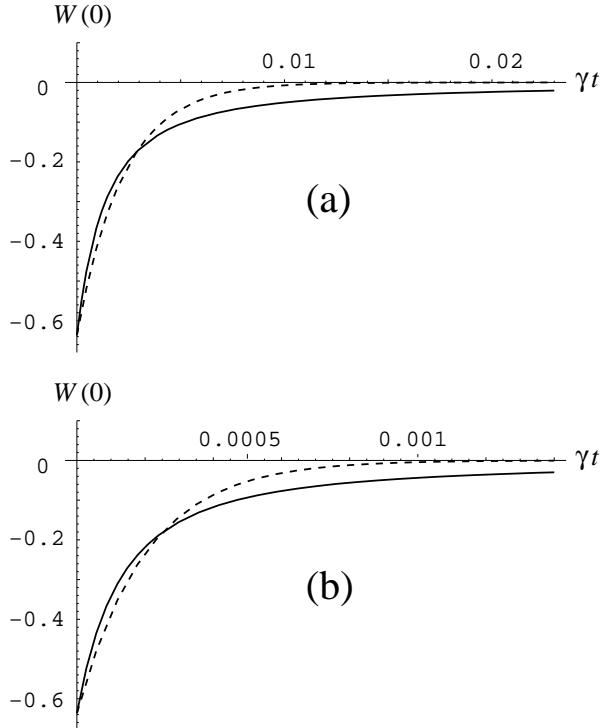


FIG. 2: The minimum negative values of the Wigner function of pure MQSs (dashed curves) and of highly-mixed MQSs (solid curves) for the same separation (i.e. the same α) between the component states. (a) A pure MQS with $\alpha = 30$, and a highly-mixed MQS with $V = 10^3$ and $\alpha = 30$. The average photon number of the pure MQS is 900 while that of the highly-mixed MQS is $\approx 1.4 \times 10^3$. (b) A pure MQS with $\alpha = 100$, and a highly-mixed MQS with $V = 10^4$ and $\alpha = 100$. The average photon number of the pure MQS is 10^4 while that of the highly-mixed MQS is $\approx 1.5 \times 10^4$. In both cases, the minimum negative values of pure MQSs approach zero faster than those of highly-mixed MQSs after a certain time.

ranging terms for the mixed MQS in Eq. (4) to give

$$\rho = \int d^2\beta P_{\{V,\alpha\}}(\beta) |\Psi_\beta\rangle\langle\Psi_\beta|, \quad (14)$$

where

$$P_{\{V,\alpha\}}(\beta) = \frac{2}{\pi(V-1)} \frac{1-e^{-2|\beta|^2}}{1-\frac{1}{V}e^{-\frac{2}{V}|\alpha|^2}} e^{-\frac{2}{V-1}|\beta-\alpha|^2}, \quad (15)$$

and the state $|\Psi_\beta\rangle = N_\beta (|\beta\rangle - |-\beta\rangle)$ is the normalized pure MQS (β : complex). Now, the mixed MQS in Eq. (14) may be interpreted as the classical mixture of the pure MQS, $|\Psi_\beta\rangle$, with $P_{\{V,\alpha\}}(\beta)$ as the weighting function. The quantum interference of each component state $|\Psi_\beta\rangle$ will decay at the rate of $2\gamma|\beta|^2$, as $e^{-2\gamma|\beta|^2 t}$ [11]. As the probability distribution $P_{\{V,\alpha\}}(\beta)$ is centered at α with the variance determined by V in the phase space, the mixed state ρ is composed of both of the faster and the slower decaying pure states than the single pure coherent superposition $|\Psi_\alpha\rangle$. In the long-time limit, only

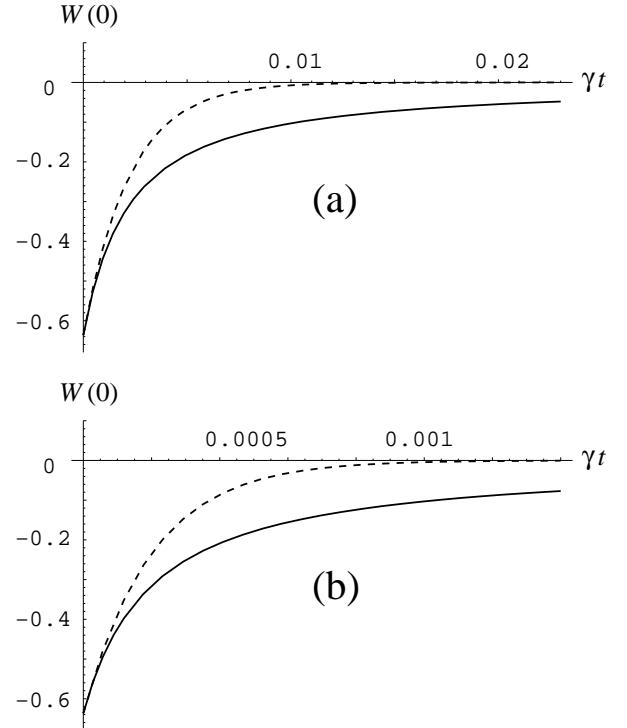


FIG. 3: The minimum negative values for the Wigner functions of pure MQSs (dashed curves) and highly-mixed MQSs (solid curves) for the same average photon numbers: (a) a pure MQS with $\alpha = 30$, and a highly-mixed MQS with $V = 10^3$ and $\alpha = 20$, where the average photon number of each state is equally ≈ 900 , and (b) a pure MQS with $\alpha \approx 100$, and a highly-mixed MQS with $V = 1.5 \times 10^4$ and $\alpha = 50$, where the average photon number of each state is equally $\approx 1.0 \times 10^4$. The minimum negative values of the pure MQSs obviously approach zero faster than those of the highly-mixed MQSs.

the slower decaying states will survive, which constitutes the basis for the overall slower decoherence of the mixed MQS in Fig. 2. The difference in decoherence between the mixed MQS and the pure MQS in Fig. 3 can also be similarly explained.

To illustrate the present situation, let us consider the mixture of decaying terms $e^{-2\gamma|\beta|^2 t}$ with the probability density $P_{\{V,\alpha\}}(\beta)$, that is,

$$\begin{aligned} C(t) &= \int d^2\beta P_{\{V,\alpha\}}(\beta) e^{-2\gamma|\beta|^2 t} \\ &= \frac{1}{1-\frac{1}{V}e^{-\frac{2}{V}|\alpha|^2}} [D(\gamma t) - D(\gamma t + 1)], \end{aligned} \quad (16)$$

where

$$D(x) \equiv \frac{1}{1+(V-1)x} e^{-\frac{2|\alpha|^2 x}{1+(V-1)x}}. \quad (17)$$

In Fig. 4, we plot $C(t)$ with the same values of α and V as the ones in Fig. 2. Although $C(t)$ does not necessarily represent the same physical context as the Wigner

IV. CONCLUSIONS

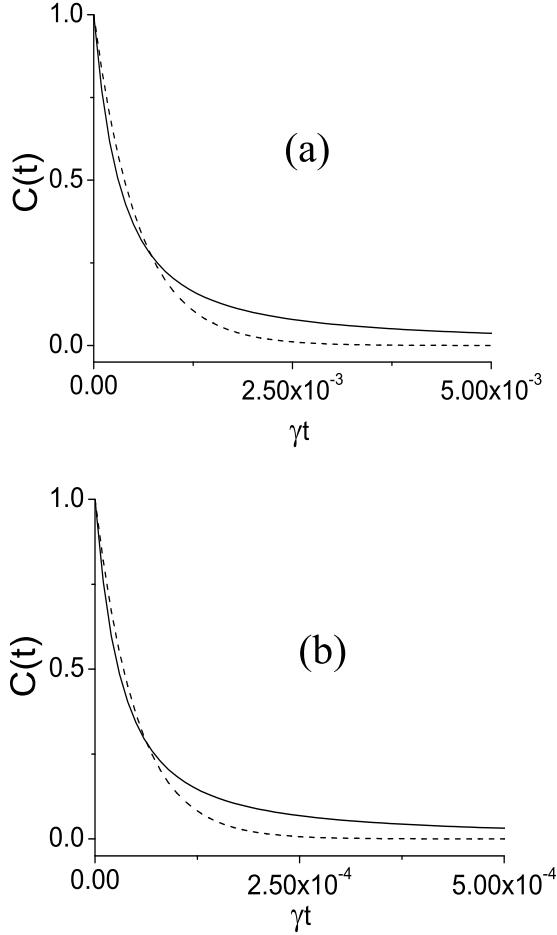


FIG. 4: The probabilistic mixture $C(t)$ in Eq. (16) (solid curves) of exponentially decaying terms, as compared with the single decaying term $e^{-2\gamma|\alpha|^2t}$ (dashed curves), for (a) $\alpha = 30$, $V = 10^3$, and (b) $\alpha = 100$, $V = 10^4$.

function $W(0)$ over time, the plot reveals a similar trait: $C(t)$ decays faster in early time and slower in long time than the single decay term $e^{-2\gamma|\alpha|^2t}$.

In this paper, we have compared decoherence evolutions of pure MQSs and mixed MQSs, and our study has revealed previously unknown aspects of decoherence phenomena. In particular, we have found that a highly mixed MQS (actual state) can show significant nonclassicality even at times when the corresponding pure MQS (target state) loses its quantum property almost completely. The mixed MQS is more macroscopic in view of energy quanta (mean photon number), and our result thus implies that the energy size of the macroscopic system may not be a good indicator of the decoherence rate beyond the very short time regime. This has been further evidenced by comparing the pure and the mixed MQS having the same energy quanta in Fig. 3. Note that the mean photon number \bar{n} of the system decays as $\bar{n} = \bar{n}_0 e^{-\gamma t}$ from the master eq. (7), thus it takes the same value at all times for both of the pure and the mixed MQS in Fig. 3.

Our findings also have a practical implication: In an optics experiment to generate a MQS, although our target state is a pure MQS, the actual output state will be in most cases a mixed MQS due to the inevitable experimental noise added to the initial state. The nonclassicality of a MQS, however, can be still observable in moderate times after it is exposed to an environment. Although the noise was modeled as a Gaussian one in this paper, the main result does not change for different types of noise, as long as the initial mixed MQS possesses as its components the pure MQS's that are more robust against decoherence than the single pure MQS. Decoherence properties of two-mode MQSs [24], in relation to quantum entanglement and quantum nonlocality, deserve further investigations.

Acknowledgments

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